PHYSICAL REVIEW A **78**, 010305(R) (2008)

Maximally entangling tripartite protocols for Josephson phase qubits

Andrei Galiautdinov*
Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30602, USA

John M. Martinis†
Department of Physics, University of California, Santa Barbara, California 93106, USA

(Received 15 April 2008; published 9 July 2008)

We introduce a suit of simple entangling protocols for generating tripartite Greenberger-Horne-Zeilinger and W states in systems with anisotropic exchange interaction $g(XX+YY)+\tilde{g}ZZ$. An interesting example is provided by macroscopic entanglement in Josephson phase qubits with capacitive ($\tilde{g}=0$) and inductive ($0<|\tilde{g}/g|<0.1$) couplings.

DOI: 10.1103/PhysRevA.78.010305 PACS number(s): 03.67.Bg, 03.67.Lx, 85.25.−j

I. INTRODUCTION

Superconducting circuits with Josephson junctions have attracted considerable attention as promising candidates for scalable solid-state quantum computing architectures [1]. The story began in the early 1980s, when Tony Leggett made a remarkable prediction that under certain experimental conditions the macroscopic variables describing such circuits could exhibit a characteristically quantum behavior [2]. Several years later, such behavior was unambiguously observed in a series of tunneling experiments by Devoret *et al.* [3], Martinis *et al.* [4], and Clarke *et al.* [5]. It was eventually realized that due to their intrinsic anharmonicity, the ease of manipulation, and relatively long coherence times [6], the metastable macroscopic quantum states of the junctions could be used as the states of the qubits. That idea had recently been supported by successful experimental demonstrations of Rabi oscillations [7], high-fidelity state preparation and measurement [8–13], and various logic gate operations [9–12,14]. Further progress in developing a workable quantum computer will depend on the architecture's ability to generate various multiqubit entangled states that form the basis for many important information-processing algorithms [15].

In this paper, we develop several *single-step entangling* protocols suitable for generating maximally entangled quantum states in tripartite systems with pairwise coupling $g(XX+YY)+\tilde{g}ZZ$. We base our approach on the idea that implementing symmetric states may conveniently be done by symmetrical control of all the qubits in the system. This bears a resemblance to approaches routinely used in digital electronics: while an arbitrary gate (for example, a three-bit gate) can be made from a collection of NAND gates, it is often convenient to use more complicated designs with three input logic gates to make the needed gate faster and/or smaller.

The protocols developed in this paper may be directly applied to virtually any of the currently known superconducting qubit architectures described [in the rotating wave approximation (RWA)] by the Hamiltonians of the form [16]

$$H_{\mathrm{RWA}} = (1/2)[\vec{\Omega}_1 \cdot \vec{\sigma}_1 + \vec{\Omega}_2 \cdot \vec{\sigma}_2 + g(\sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2) + \tilde{g}\sigma_z^1\sigma_z^2], \tag{1}$$

with either $\tilde{g}=0$ (capacitive coupling case) or $0<|\tilde{g}/g|<0.1$ (inductive coupling case) [17].

Recall that in the RWA, an off-resonance "counter-rotating" term is ignored in the dynamics. This is typically a good approximation for experiments with superconducting qubits because the time to do an operation (∼10 ns) is much slower than the inverse time scale of the qubit transition, ∼0.1 ns. These time scales give an amplitude error from the counter-rotating drive of order 1/100 and a probability error of order 10^{-4}. Most theories for qubit logic gates make this approximation.

II. THE GREENBERGER-HORNE-ZEILINGER (GHZ) PROTOCOL

A. Triangular coupling scheme

In the rotating frame (interaction picture) in the absence of coupling, the system's Hamiltonian is represented by a zero matrix, and thus all computational basis states $|000\rangle$, $|001\rangle$, $|010\rangle$, $|100\rangle$, $|011\rangle$, $|101\rangle$, $|110\rangle$, $|111\rangle$ have the same effective energy $E_{\mathrm{eff}}=0$ (no time evolution). The pairwise coupling $H_{\mathrm{int}}=(1/2)\Sigma_{i=1}^3[g(\sigma_x^i\sigma_x^{i+1}+\sigma_y^i\sigma_y^{i+1})+\tilde{g}\sigma_z^i\sigma_z^{i+1}]$ *partially lifts* the degeneracy, which results in the new energy spectrum,

$$E_{\mathrm{int}} = \{3\tilde{g}/2, 3\tilde{g}/2, 2g-\tilde{g}/2, 2g-\tilde{g}/2, -(g+\tilde{g}/2), -(g+\tilde{g}/2), -(g+\tilde{g}/2), -(g+\tilde{g}/2)\}, \tag{2}$$

and the corresponding $\mathcal{H}$ eigenbasis,

$$\mathcal{H}_{\mathrm{GHZ}} \oplus \mathcal{H}_{\mathrm{W}} \oplus \mathcal{H}_{\mathrm{rest}} \equiv \{|000\rangle \oplus |111\rangle\} \oplus \{|\mathrm{W}\rangle \oplus |\mathrm{W}'\rangle\} \oplus \{|\Psi_1\rangle \oplus |\Psi_1{}'\rangle \oplus |\Psi_2\rangle \oplus |\Psi_2{}'\rangle\}, \tag{3}$$

where

$$|\mathrm{W}\rangle = (|100\rangle + |010\rangle + |001\rangle)/\sqrt{3},$$

$$|\mathrm{W}'\rangle = (|011\rangle + |101\rangle + |110\rangle)/\sqrt{3},$$

*ag@physast.uga.edu
†martinis@physics.ucsb.edu

$$|\Psi_1\rangle = (|100\rangle - |010\rangle)/\sqrt{2}, \quad |\Psi_1{}'\rangle = (|011\rangle - |101\rangle)/\sqrt{2},$$

$$|\Psi_2\rangle = (|100\rangle + |010\rangle - 2|001\rangle)/\sqrt{6},$$

$$|\Psi_2{}'\rangle = (|011\rangle + |101\rangle - 2|110\rangle)/\sqrt{6}. \quad (4)$$

Since the coupling does not cause transitions within each of the degenerate subspaces (nor does it cause transitions between different such subspaces), it is impossible to generate the $|\mathrm{GHZ}\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ state from the ground state $|000\rangle$ by direct application of H_{int}. Instead, we must first bring the $|000\rangle$ state out of the $\mathcal{H}_{\mathrm{GHZ}}$ subspace by, for example, subjecting it to a local rotation R_1 in such a way as to produce a state $|\psi\rangle$ that has both $|000\rangle$ and $|111\rangle$ components. That is only possible if *all* one-qubit amplitudes $\alpha_1, \ldots, \beta_3$ in the resulting product state $|\psi\rangle = R_1|000\rangle = (\alpha_1|0\rangle + \beta_1|1\rangle)(\alpha_2|0\rangle + \beta_2|1\rangle)(\alpha_3|0\rangle + \beta_3|1\rangle)$ are chosen to be nonzero, which means that in the computational basis the state $|\psi\rangle$ will have eight nonzero components.

We now notice that in the $\mathcal{H}$ basis, the three-qubit rotations are block-diagonal,

$$X_\theta = X_\theta^{(3)} X_\theta^{(2)} X_\theta^{(1)} = \begin{pmatrix} c^3 & is^3 & -i\sqrt{3}sc^2 & -\sqrt{3}cs^2 \\ is^3 & c^3 & -\sqrt{3}cs^2 & -i\sqrt{3}sc^2 \\ -i\sqrt{3}sc^2 & -\sqrt{3}cs^2 & c(1-3s^2) & is(1-3c^2) \\ -\sqrt{3}cs^2 & -i\sqrt{3}sc^2 & is(1-3c^2) & c(1-3s^2) \end{pmatrix} \oplus \begin{pmatrix} c & is \\ is & c \end{pmatrix} \oplus \begin{pmatrix} c & is \\ is & c \end{pmatrix},$$

$$Y_\theta = Y_\theta^{(3)} Y_\theta^{(2)} Y_\theta^{(1)} = \begin{pmatrix} c^3 & -s^3 & -\sqrt{3}sc^2 & \sqrt{3}cs^2 \\ s^3 & c^3 & \sqrt{3}cs^2 & \sqrt{3}sc^2 \\ \sqrt{3}sc^2 & \sqrt{3}cs^2 & c(1-3s^2) & s(1-3c^2) \\ \sqrt{3}cs^2 & -\sqrt{3}sc^2 & -s(1-3c^2) & c(1-3s^2) \end{pmatrix} \oplus \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \oplus \begin{pmatrix} c & s \\ -s & c \end{pmatrix}, \quad (5)$$

where $X_\theta^{(k)} = \exp(-i\theta\sigma_x^k/2)$, $Y_\theta^{(k)} = \exp(-i\theta\sigma_y^k/2)$, $k=1,2,3$, and $c \equiv \cos(\theta/2)$ and $s \equiv \sin(\theta/2)$. For $\theta = \pi/2$, the corresponding 4×4 blocks acting on the $\mathcal{H}_{\mathrm{GHZ}} \oplus \mathcal{H}_{\mathrm{W}}$ subspace are

$$X_{\pi/2}^{(4\times4)} = \frac{1}{\sqrt{8}} \begin{pmatrix} 1 & i & -i\sqrt{3} & -\sqrt{3} \\ i & 1 & -\sqrt{3} & -i\sqrt{3} \\ -i\sqrt{3} & -\sqrt{3} & -1 & -i \\ -\sqrt{3} & -i\sqrt{3} & -i & -1 \end{pmatrix},$$

$$Y_{\pi/2}^{(4\times4)} = \frac{1}{\sqrt{8}} \begin{pmatrix} 1 & -1 & -\sqrt{3} & \sqrt{3} \\ 1 & 1 & \sqrt{3} & \sqrt{3} \\ \sqrt{3} & \sqrt{3} & -1 & -1 \\ \sqrt{3} & -\sqrt{3} & 1 & -1 \end{pmatrix}. \quad (6)$$

This shows that $Y_{\pi/2}$ provides a convenient choice for R_1. We may thus start by generating the so-called uniform superposition state,

$$|\psi\rangle_{\mathrm{uniform}} \equiv (1/\sqrt{8})(|000\rangle + |001\rangle + \cdots + |110\rangle + |111\rangle) = Y_{\pi/2}|000\rangle = (1/2)[|\mathrm{GHZ}\rangle + \sqrt{3/2}(|\mathrm{W}\rangle + |\mathrm{W}'\rangle)] \in \mathcal{H}_{\mathrm{GHZ}} \oplus \mathcal{H}_{\mathrm{W}}. \quad (7)$$

The entanglement is then performed by acting on $|\psi\rangle_{\mathrm{uniform}}$ with $U_{\mathrm{int}} = \exp(-iH_{\mathrm{int}}t)$, thus inducing a phase difference between the GHZ and $\mathrm{W}+\mathrm{W}'$ components (this step works only for $g \neq \tilde{g}$, see Sec. IV),

$$U_{\mathrm{int}} Y_{\pi/2}|000\rangle = (e^{-i\alpha}/2)[|\mathrm{GHZ}\rangle + e^{-i\delta}\sqrt{3/2}(|\mathrm{W}\rangle + |\mathrm{W}'\rangle)],$$

$$\alpha = (3\tilde{g}/2)t, \quad \delta = 2(g - \tilde{g})t. \quad (8)$$

To transform to the desired GHZ state, we first diagonalize the $X_{\pi/2}^{(4\times4)}$ and $Y_{\pi/2}^{(4\times4)}$ operators to get the unimodular spectra,

$$\lambda_X = \{-e^{i(\pi/4)}, -e^{-i(\pi/4)}, e^{-i(\pi/4)}, e^{i(\pi/4)}\},$$

$$\lambda_Y = \{-e^{-i(\pi/4)}, -e^{i(\pi/4)}, e^{i(\pi/4)}, e^{-i(\pi/4)}\}, \quad (9)$$

and the eigenbases $\mathcal{X} = \{|X_1\rangle, |X_2\rangle, |X_3\rangle, |X_4\rangle\}$ and $\mathcal{Y} = \{|Y_1\rangle, |Y_2\rangle, |Y_3\rangle, |Y_4\rangle\}$, whose vectors are given by the columns of $Y_{\pi/2}^{(4\times4)}$ and $X_{\pi/2}^{(4\times4)}$, correspondingly.

Using the $\mathcal{X}$ basis, we notice that both states

$$|\mathrm{GHZ}\rangle = \frac{|X_1\rangle + \sqrt{3}|X_4\rangle}{2},$$

$$U_{\mathrm{int}} Y_{\pi/2}|000\rangle = \frac{e^{-i\alpha}}{2}\left(\frac{1+3e^{-i\delta}}{2}|X_1\rangle + \frac{1-e^{-i\delta}}{2}\sqrt{3}|X_4\rangle\right), \quad (10)$$

belong to the same two-dimensional subspace $|X_1\rangle \oplus |X_4\rangle$. Therefore, by performing an additional $X_{\pi/2}$ rotation, we can transform $U_{\mathrm{int}} Y_{\pi/2}|000\rangle$ to

$$X_{\pi/2} U_{\mathrm{int}} Y_{\pi/2}|000\rangle = e^{-i\alpha} e^{i(\pi/4)}|\mathrm{GHZ}\rangle, \quad (11)$$

provided the entangling time is set to give $|\delta| = \pi$, or $t_{\mathrm{GHZ}} = \pi/2|g - \tilde{g}|$. Any other GHZ state $(|000\rangle + e^{i\phi}|111\rangle)/\sqrt{2}$ can be made out of the "standard" GHZ state by a Z rotation applied to *one* of the qubits, as usual.

The protocol may be compared to controlled-NOT logic gate implementations [16] that used various sequences $R_2 U_{\mathrm{CNOT}} R_1 = e^{i(\pi/4)}\mathrm{CNOT}$, $\det(U_{\mathrm{CNOT}}) = +1$, with (entangling) times $t_{\mathrm{CNOT}} = T(\pi/2g)$, $1 \leqslant T < 1.6$. Thus, for $\tilde{g} = 0$, the entangling operation proposed here will be of same duration as the fastest possible CNOT.

We conclude this section by noting that in its present form, the GHZ protocol *cannot* be used to generate the W state. This can be seen by writing $|\mathrm{W}\rangle = [\sqrt{3}(|X_1\rangle + |X_2\rangle)$

$-(|X_3\rangle+|X_4\rangle)]/\sqrt{8}$, which shows that our $XU_{\text{int}}Y$ sequence does not result in a W since the final $X_{\pi/2}$ rotation cannot eliminate the $|X_2\rangle$ and $|X_3\rangle$ components. Also,

$$|\mathrm{W}\rangle=[\sqrt{3}(i|Y_1\rangle-|Y_2\rangle)-(|Y_3\rangle-i|Y_4\rangle)]/\sqrt{8} \tag{12}$$

and

$$Y_{\pi/2}U_{\text{int}}Y_{\pi/2}|000\rangle=e^{-i\alpha}\left(\frac{1-3e^{-i\delta}}{2}(i|Y_1\rangle-|Y_2\rangle)-\frac{\sqrt{3}(1+e^{-i\delta})}{2}(|Y_3\rangle-i|Y_4\rangle)\right)\Big/\sqrt{8}, \tag{13}$$

and thus no choice of δ will work for the $YU_{\text{int}}Y$ sequence either.

B. Linear-coupling scheme

In the case of linear coupling, say $1\leftrightarrow 2$ and $2\leftrightarrow 3$, the energy spectrum is given by

$$E_{\text{int}}=\{\tilde{g},\tilde{g},\epsilon^{(+)},\epsilon^{(+)},\epsilon^{(-)},\epsilon^{(-)},0,0\},$$

$$\varepsilon^{(\pm)}=\pm\sqrt{2g^2+(\tilde{g}/2)^2}-\tilde{g}/2, \tag{14}$$

with eigenbasis

$$|000\rangle,\quad |111\rangle,$$

$$|\mathrm{W}\rangle^{(+)}=C^{(+)}[|001\rangle+(\epsilon^{(+)}/g)|010\rangle+|001\rangle],$$

$$|\mathrm{W}'\rangle^{(+)}=C^{(+)}[|011\rangle+(\epsilon^{(+)}/g)|101\rangle+|110\rangle],$$

$$|\mathrm{W}\rangle^{(-)}=C^{(-)}[|001\rangle+(\epsilon^{(-)}/g)|010\rangle+|001\rangle],$$

$$|\mathrm{W}'\rangle^{(-)}=C^{(-)}[|011\rangle+(\epsilon^{(-)}/g)|101\rangle+|110\rangle],$$

$$|\Psi\rangle=(|001\rangle-|100\rangle)/\sqrt{2},\quad |\Psi'\rangle=(|011\rangle-|110\rangle)/\sqrt{2}, \tag{15}$$

where $C^{(\pm)}$ are normalizing constants. We have,

$$|\mathrm{W}\rangle=A^{(+)}|\mathrm{W}\rangle^{(+)}+A^{(-)}|\mathrm{W}\rangle^{(-)},$$

$$A^{(+)}=\frac{1}{C^{(+)}}\frac{-\epsilon^{(-)}+g}{\epsilon^{(+)}-\epsilon^{(-)}},\quad A^{(-)}=\frac{1}{C^{(-)}}\frac{\epsilon^{(+)}-g}{\epsilon^{(+)}-\epsilon^{(-)}}, \tag{16}$$

and similarly for $|\mathrm{W}'\rangle$. Our GHZ sequence then leads to the entangled state

$$U_{\text{int}}Y_{\pi/2}|000\rangle=(e^{-i\alpha}/2)\{|\mathrm{GHZ}\rangle+\sqrt{3/2}(e^{-i\delta^{(+)}}A^{(+)}[|\mathrm{W}\rangle^{(+)}+|\mathrm{W}'\rangle^{(+)}]+e^{-i\delta^{(-)}}A^{(-)}[|\mathrm{W}\rangle^{(-)}+|\mathrm{W}'\rangle^{(-)}])\}, \tag{17}$$

with $\alpha=\tilde{g}t$, $\delta^{(\pm)}=(\epsilon^{(\pm)}-\tilde{g})t$. Since $t>0$, in order for the $X_{\pi/2}$ post-rotation to give a GHZ, we must restrict coupling to $\tilde{g}=0$ and set the entangling time to $t_{\text{GHZ}}=\pi/\sqrt{2}|g|$. An alternative GHZ implementation for superconducting qubit systems with capacitive coupling has recently been considered [18]. There, individual qubits were conditionally operated upon one at a time.

III. THE W PROTOCOL

We now turn to the W protocol. Equation (16) suggests that control sequence $YU_{\text{int}}Y$ may still give a W, provided a proper adjustment of $i|Y_1\rangle-|Y_2\rangle$ and $|Y_3\rangle-i|Y_4\rangle$ amplitudes is made by a physically acceptable change of the system's Hamiltonian. In the context of Josephson phase qubits, such modification can be achieved by adding local Rabi term(s) to H_{int}, for instance, $H_{\text{int}}^{\Omega}=(\Omega/2)(\sigma_x^1+\sigma_x^2+\sigma_x^3)+H_{\text{int}}$. The energy spectrum then becomes

$$E_{\text{int}}^{\Omega}=\{\epsilon^{(+)}\pm\chi^{(+)},\epsilon^{(-)}\pm\chi^{(-)},-\epsilon^{(+)},-\epsilon^{(+)},-\epsilon^{(-)},-\epsilon^{(-)}\}, \tag{18}$$

with

$$\epsilon^{(\pm)}=g+\tilde{g}/2\pm\Omega/2,\quad \chi^{(\pm)}=\sqrt{(g-\tilde{g})^2\pm(g-\tilde{g})\Omega+\Omega^2}. \tag{19}$$

The (first two) eigenvectors are

$$|\Phi_{1,2}^{(+)}\rangle=C_{1,2}^{(+)}\{[-1-(2/\Omega)(g-\tilde{g}\mp\chi^{(+)})]|\mathrm{GHZ}\rangle+\sqrt{3/2}(|\mathrm{W}\rangle+|\mathrm{W}'\rangle)\}, \tag{20}$$

with normalizing constants $C_k^{(+)}$, $k=1,2$. After some algebra, we find

$$U_{\text{int}}^{\Omega}Y_{\pi/2}|000\rangle=e^{-i\alpha}/(4\sqrt{2}\chi^{(+)})[(A/\Omega)(ie^{i(\pi/4)}|Y_1\rangle-e^{-i(\pi/4)}\times|Y_2\rangle)+(\sqrt{3}B/\Omega)(e^{-i(\pi/4)}|Y_3\rangle-ie^{i(\pi/4)}|Y_4\rangle)], \tag{21}$$

where

$$A=(g-\tilde{g}+\Omega+\chi^{(+)})(g-\tilde{g}+2\Omega-\chi^{(+)})-e^{-i\delta}(g-\tilde{g}+\Omega-\chi^{(+)})(g-\tilde{g}+2\Omega+\chi^{(+)}),$$

$$B=(g-\tilde{g}+\Omega+\chi^{(+)})(g-\tilde{g}-\chi^{(+)})-e^{-i\delta}(g-\tilde{g}+\Omega-\chi^{(+)})(g-\tilde{g}+\chi^{(+)}), \tag{22}$$

and $\alpha=(\epsilon^{(+)}+\chi^{(+)})t$, $\delta=-2\chi^{(+)}t$. It is straightforward to verify that additional $Y_{\pi/2}$ rotation applied to this state produces a W [see Eqs. (9) and (12)],

$$Y_{\pi/2}U_{\text{int}}^{\Omega}Y_{\pi/2}|000\rangle=[-\operatorname{sgn}(g-\tilde{g})]e^{-i\alpha}|\mathrm{W}\rangle, \tag{23}$$

provided we set $t_{\mathrm{W}}=\pi/\sqrt{3}|g-\tilde{g}|$, $\Omega=-(g-\tilde{g})/2$.

IV. ADDENDUM: ISOTROPIC HEISENBERG EXCHANGE $g(XX+YY+ZZ)$

Maximally entangling protocols introduced in the previous sections are singular in the limit $\tilde{g}\to g$, which corresponds to the isotropic Heisenberg exchange interaction. Even though this limit is not met in superconducting qubits, for completeness we briefly discuss it here.

It is obvious that when $g=\tilde{g}$, the uniform state $Y_{\pi/2}|000\rangle$ is an eigenstate of the interaction Hamiltonian. Consequently, the Heisenberg exchange does not cause transitions out of it, making the gate time divergent. To perform single-step entanglement, we break the symmetry of the local rotations. For example, the GHZ state can be generated by

$$e^{-i\alpha}|\mathrm{GHZ}\rangle = e^{-i(\pi/2)\sigma_z^2}e^{-i(\pi/3)(\sigma_y^1-\sigma_y^2)} \times U_{\mathrm{int}}e^{-i(\pi/12)(5\sigma_y^1+\sigma_y^2-3\sigma_y^3)}e^{-i(\pi/2)\sigma_z^2}|000\rangle,$$

$$\alpha = -\pi/2, \quad t_{\mathrm{GHZ}} = (2/3)(\pi/2g). \tag{24}$$

To generate the W state, we generalize Neeley's fast implementation [19] for triangular $g(XX+YY)$ coupling to arbitrary coupling $g(XX+YY)+\tilde{g}ZZ$, including the Heisenberg exchange $g=\tilde{g}$,

$$e^{-i\alpha}|\mathrm{W}\rangle = e^{+i(\pi/3)\sigma_z^2}U_{\mathrm{int}}e^{-i(\pi/2)\sigma_y^2}|000\rangle,$$

$$\alpha = (5g-2\tilde{g})\pi/18g, \quad t_{\mathrm{W}} = (4/9)(\pi/2g). \tag{25}$$

V. CONCLUSION

In summary, we have developed several single-step *symmetric* implementations for generating maximally entangled tripartite quantum states in systems with anisotropic exchange interaction that are directly applicable to superconducting qubit architectures. In the GHZ case, both triangular and linear-coupling schemes have been analyzed. In the isotropic limit, our implementations exhibit singularities that can be removed by breaking the symmetry of the local pulses.

ACKNOWLEDGMENTS

This work was supported by the DTO under Grant No. W911NF-04-1-0204 and by the NSF under Grants No. CMS-0404031 and No. CCF-0507227. The authors thank Michael Geller and Matthew Neeley for helpful discussions.

[1] See, for example, Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001); J. Q. You and F. Nori, Phys. Today **58** (11), 42 (2005).

[2] A. J. Leggett, Suppl. Prog. Theor. Phys. **69**, 80 (1980).

[3] M. H. Devoret, J. M. Martinis, and J. Clarke, Phys. Rev. Lett. **55**, 1908 (1985).

[4] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B **35**, 4682 (1987).

[5] J. Clarke *et al.*, Science **239**, 992 (1988).

[6] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002); S. Han, Y. Yu, Xi Chu, S. Chu, and Z. Wang, *ibid.* **293**, 1457 (2001); Y. Yu, S. Han, X. Chu, S. Chu, and Z. Wang, *ibid.* **296**, 889 (2002).

[7] Y. Nakamura, C. D. Chen, and J. S. Tsai, Phys. Rev. Lett. **79**, 2328 (1997); Y. Nakamura, Y. A. Pashkin, T. Yamamoto, and J. S. Tsai, *ibid.* **88**, 047901 (2002).

[8] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R. S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature **431**, 162 (2004); O. Astafiev, Y. A. Pashkin, T. Yamamoto, Y. Nakamura, and J. S. Tsai, Phys. Rev. B **69**, 180507(R) (2004); I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, Science **299**, 1869 (2003); B. L. T. Plourde, T. L. Robertson, P. A. Reichardt, T. Hime, S. Linzen, C.-E. Wu, and J. Clarke, Phys. Rev. B **72**, 060506(R) (2005); J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, Phys. Rev. Lett. **89**, 117901 (2002).

[9] R. W. Simmonds, K. M. Lang, D. A. Hite, S. Nam, D. P. Pappas, and J. M. Martinis, Phys. Rev. Lett. **93**, 077003 (2004).

[10] K. B. Cooper, M. Steffen, R. McDermott, R. W. Simmonds, S. Oh, D. A. Hite, D. P. Pappas, and J. M. Martinis, Phys. Rev. Lett. **93**, 180401 (2004).

[11] J. Claudon, F. Balestro, F. W. J. Hekking, and O. Buisson, Phys. Rev. Lett. **93**, 187003 (2004); J. Claudon, A. Fay, L. P. Levy, and O. Buisson, Phys. Rev. B **73**, 180502(R) (2006).

[12] N. Katz, M. Ansmann, R. C. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. M. Weig, A. N. Cleland, J. M. Martinis, and A. N. Korotkov, Science **312**, 1498 (2006).

[13] Y. X. Liu, L. F. Wei, and F. Nori, Phys. Rev. B **72**, 014547 (2005); R. McDermott, R. W. Simmonds, M. Steffen, K. B. Cooper, K. Cicak, K. D. Osborn, S. Oh, D. P. Pappas, and J. M. Martinis, Science **307**, 1299 (2005); M. Steffen, M. Ansmann, R. C. Bialczak, N. Katz, E. Lucero, R. McDermott, M. Neeley, E. M. Weig, A. N. Cleland, and J. M. Martinis, *ibid.* **313**, 1423 (2006).

[14] P. R. Johnson, F. W. Strauch, A. J. Dragt, R. C. Ramos, C. J. Lobb, J. R. Anderson, and F. C. Wellstood, Phys. Rev. B **67**, 020509(R) (2003).

[15] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).

[16] A. Galiautdinov, Phys. Rev. A **75**, 052303 (2007); J. Math. Phys. **48**, 112105 (2007).

[17] M. Geller (private communication).

[18] L. F. Wei, Yu-xi Liu, and F. Nori, Phys. Rev. Lett. **96**, 246803 (2006); Sh. Matsuo, S. Ashhab, T. Fujii, F. Nori, K. Nagai, and N. Hatakenaka, J. Phys. Soc. Jpn. **76**, 054802 (2007).

[19] M. Neeley (private communication).